DICK SCHAFF MATH SUPERBOWL XLIII
Level 5: Precalculus Blitz – 2016

Directions: Select the most correct answer for each question and mark it on the Scantron® sheet. Note that N.O.T. means "None of These."

1. Find all the values of x for which $\frac{x}{2x+4} \geq 1$.

(A) $(2, \infty)$ (B) $[0, 2)$ (C) $[-4, -2)$ (D) $[-2, 2)$ (E) N.O.T.

2. Find all values of k such that the equation $\log(kx) = 2\log(x + 9)$ has exactly one real solution.

(A) $k < 0$ (B) $k = 36$ (C) $k = 9$ (D) A or B (E) N.O.T.

3. What is the linear velocity in inches per second of a tip of a 20-inch lawn mower blade that is rotating at 3,000 revolutions per minute?

(A) 600 (B) 1000π (C) 1000 (D) 300π (E) N.O.T.

4. A radioactive isotope has a half-life of 450 years. How many years will it take for a sample of the isotope to decrease from 6 mg to 2 mg?

(A) $\frac{450 \ln 3}{\ln 2}$ (B) $\frac{450 \ln 3}{-\ln 2}$ (C) $\frac{\ln 3}{450 \ln 2}$ (D) $\frac{\ln 2}{450 \ln 3}$ (E) N.O.T.

5. Find the circumference, in terms of π, of a circle whose ratio of area to circumference is 2016.

(A) 1008π (B) 2016π (C) 4032π (D) 8064π (E) N.O.T.

6. If $f(x) = \frac{1}{x} + 1$, then $(f \circ f)(x) =$

(A) $\frac{2x+1}{x+1}$ (B) $\frac{1}{x^2} + 1$ (C) $\frac{2}{x} + 2$ (D) $\frac{1}{x^2} + \frac{2}{x} + 1$ (E) N.O.T.

7. Let a + b = M and ab = N. The polynomial $a^4 + b^4$ can be represented as $PM^4 - QM^2N + RN^2$, where P, Q, and R are positive integers. Find P + Q + R.

(A) 3 (B) 4 (C) 5 (D) 6 (E) N.O.T.

8. Find $\sin^{-1}(\sin(18\pi/5))$.

(A) $\pi/5$ (B) $2\pi/5$ (C) $-3\pi/5$ (D) $-2\pi/5$ (E) N.O.T.

9. Find $\tan(\sin^{-1}(5/13))$.

(A) 1/4 (B) 12/5 (C) 12/13 (D) 1/3 (E) N.O.T.

10. $(\sin t + \cos t)^2 =$

(A) $2\sin t \cos t + 1$ (B) 1

(C) $1 + 2\sin t$ (D) $\sin(t^2) + \cos(t^2)$ (E) N.O.T.

11. What is the remainder of $x+a\overline{)x^4+a^2}$?

(A) $a^2 + a^4$ (B) a^6 (C) $a^2 - a^4$ (D) a^8 (E) N.O.T.

12. Find all the values of x for which $\leq \frac{25}{x}$.

(A) $(-\infty, -5] \cup (0,5)$ (B) $(-\infty, -5] \cup [0,5]$

(C) $(-\infty, -5) \cup [0,5]$ (D) $(-\infty, -5) \cap (0, 5]$ (E) N.O.T.

13. The domain of the function f is $\{x: -1 \leq x \leq 5\}$. If $g(x) = 2f(-x)$, what is the domain of g?

(A) $\{x: -1 \leq x \leq 5\}$ (B) $\{x: -10 \leq x \leq 2\}$

(C) $\{x: -5 \leq x \leq -1\}$ (D) $\{x: -5 \leq x \leq 1\}$ (E) N.O.T.

14. Consider the function $h(x) = \frac{x^2 e^2}{x}$. Which of the following are true about the graph of $y = h(x)$?

I. The graph has a vertical asymptote at $x = 0$.
II. The graph has a horizontal asymptote at $y = 0$.
III. The graph has a minimum point.

(A) I and II only (B) II and III only (C) I and III only (D) I, II and III (E) N.O.T.

15. The phase shift of $y = 2\cos\left(x - \frac{\pi}{3}\right)$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$ (E) N.O.T.

16. In the xy-plane, the graph of $y = x(x^2 - 2)(x^2 + x + 1)$ intersects the x-axis in how many different points?

(A) One (B) Two (C) Three (D) Four (E) N.O.T.

17. The expression $3\ln x - \ln\sqrt{x}$ is equivalent to

(A) $2\ln\sqrt{x}$ (B) $\ln(x^3 - \sqrt{x})$ (C) $\frac{\ln(x^3)}{\ln\sqrt{x}}$ (D) $\frac{5}{2}\ln x$ (E) N.O.T.

18. Which is NOT a zero of $f(x) = 2x^4 - 11x^3 + 21x^2 - 20x + 6$?

(A) 3 (B) $1 - i$ (C) $1 + i$ (D) $1/2$ (E) N.O.T.

19. What is the horizontal or oblique asymptote of $f(x) = \frac{x^3 - 4x}{2x^2 - 2}$?

(A) $y = 0$ (B) $y = \frac{1}{2}x$ (C) $y = \frac{1}{2}$ (D) $x = 1/2$ (E) N.O.T.

20. State the domain of the inverse function of $f(x) = \frac{2x-1}{3x+1}$.

(A) $\left\{x|x \neq \frac{2}{3}\right\}$ (B) $\left\{x|x \neq \frac{1}{3}\right\}$ (C) $\left\{x|x \neq -\frac{1}{2}\right\}$ (D) $\{x|x \neq 3\}$ (E) N.O.T.

21. Assume that angle A and angle B are supplementary. Which of the following is equal to $\cos A$?

(A) $\sin B$ (B) $-\sin B$ (C) $\cos B$ (D) $-\cos B$ (E) N.O.T.

22. If $f(x) = x^{2/3}$, which of the following statements is NOT true?

(A) The range of f is all real numbers.

(B) f is an even function.

(C) The graph of f is symmetric with respect to the y-axis.

(D) As $x \longrightarrow \infty$, $f(x) \longrightarrow \infty$.

(E) N.O.T.

23. What is the value of c such that $y = \csc(cx)$ has a period of 10?

(A) $\frac{7\pi}{10}$ (B) $\frac{2\pi}{5}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{5}$ (E) N.O.T.

24. You can get the graph of $y = -g(2x)$ by transforming the graph of $y = g(x)$ in the following way:

(A) Stretch horizontally and reflect across the x-axis.

(B) Stretch vertically and reflect across the y-axis.

(C) Shrink horizontally and reflect across the x-axis.

(D) Shrink horizontally and reflect across the y-axis.

(E) N.O.T.

25. If $f(x)$ is a one-to-one function, and $f(8) = 11$, then which of the following CANNOT be true?

(A) $f(11) = 8$ (B) $f^{-1}(11) = 5$ (C) $f^{-1}(5) = 3$ (D) $f^{-1}(11) = 8$ (E) N.O.T.

26. Let $f(x) = x^2$. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$.

(A) $2x + h$ (B) h (C) $2x + 1$ (D) $x + h$ (E) N.O.T.

27. Let $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{2-x}$. Find the domain of $(f \circ g)(x)$.

(A) $x > 2$ (B) $x \leq 2$ (C) $x > 0$ (D) $x < 2$ (E) N.O.T.

28. If $\theta = \sin^{-1}\left(\frac{x}{4}\right)$, then the expression $\frac{\sqrt{16-x^2}}{x}$ can be simplified to

(A) $\tan\theta$ (B) $4\tan\theta$ (C) $\cot\theta$ (D) $4\sin\theta$ (E) N.O.T.

29. Identify the conic section given by the equation: $31x^2 + 12xy + y^2 - 50x - 125 = 0$.

(A) Parabola (B) Hyperbola (C) Circle (D) Ellipse (E) N.O.T.

30. The lengths of three sides of a triangle are 4 feet, 5 feet, and 7 feet. The area (in square feet) of the triangle is

(A) 16 (B) $16\sqrt{3}$ (C) $8\sqrt{2}$ (D) $4\sqrt{6}$ (E) N.O.T.

31. If $f(x)$ is a polynomial with real coefficients and one zero of f is $6 + 7i$, where $i = \sqrt{-1}$, then another zero of f must be

(A) $-6 - 7i$ (B) $-6 + 7i$ (C) $6 + \sqrt{7}$ (D) $6 - \sqrt{7}$ (E) N.O.T.

32. An equivalent expression for $\sin(3x)$ is

(A) $3\sin x - 4\sin^3 x$ (B) $3\sin x \cos x$

(C) $-\sin x$ (D) $\sin^3 x - \cos^3 x$ (E) N.O.T.

33. Find all the rational zeros of $f(x) = 10x^3 + 53x^2 + 14x - 5$

(A) $-2, 5, -5$ (B) $-2, -5, -5$ (C) $-\frac{1}{10}, 1, -5$ (D) $-\frac{1}{2}, \frac{1}{5}, -5$ (E) N.O.T.

34. The number of students infected with the flu on a college campus after t days is modeled by the function $P(t) = \frac{200}{1+39e^{-0.3t}}$. What was the initial number of infected students?

(A) 200 (B) 10 (C) 5 (D) 39 (E) N.O.T.

35. Solve the equation: $e^x - 13 = \frac{-40}{e^x}$.

(A) -1 (B) $\ln 5, \ln 8$ (C) $\ln 4, \ln 10$ (D) $8, 5$ (E) N.O.T.

36. Does the series converge? If so, determine the limit: $36 - 12 + 4 - \frac{4}{3} + \cdots$

(A) Diverges (B) Converges; 29

(C) Converges; 28 (D) Converges; 27 (E) N.O.T.

37. Let $f(x) = \begin{cases} x^2 + 2x, & x \le 0, \\ -\frac{1}{2}x, & 0 < x \le 2, \\ -\sqrt{x-1}, & 2 < x. \end{cases}$ At which of the following points does f have a local minimum?

(A) $(2, 1)$ (B) $(-1, -1)$ (C) $(0, 0)$ (D) $(2, -1)$ (E) N.O.T.

38. Decepticorp, a multinational corporation with some… *dubious* practices, sells canisters of mutagenic goo. These canisters are right circular cylinders with a volume of 2π cm^3. What is the surface area of one of these canisters if the radius of the canister is 0.5 cm?

(A) $\frac{9}{2}\pi$ (B) $\frac{17}{2}\pi$ (C) 9π (D) $\frac{33}{2}\pi$ (E) N.O.T.

39. Compute $\lim_{k\to\infty}\left(1-\frac{2}{k}\right)^{3k}$.

(A) $\frac{1}{e^3}$ (B) e^3 (C) $\frac{1}{e^2}$ (D) e^2 (E) N.O.T.

40. A water tank is shaped like an inverted circular cone, twenty feet across and twenty feet high. How much water would this tank contain if the water was only three feet deep?

(A) $\frac{9}{2}\pi$ ft^3 (B) $\frac{27}{4}\pi$ ft^3 (C) 9π ft^3 (D) $\frac{9}{4}\pi$ ft^3 (E) N.O.T.

41. In a circle of radius 12 feet, a sector is subtended by an angle of 30°. The length (in feet) of the arc is

(A) 360 (B) $\frac{4320}{\pi}$ (C) 2π (D) 2 (E) N.O.T.

42. The area (in square feet) of the sector described in Problem #41 is:

(A) 6π (B) 2160 (C) 24π (D) 12 (E) N.O.T.

43. Find the remainder when $3x^{25}-4x^{14}+3x-2$ is divided by $x+1$.

(A) -12 (B) -2 (C) 2 (D) 12 (E) N.O.T.

44. The expression $(\log_3 7)(\log_7 11)(\log_{11} 27)$ is equivalent to

(A) $\log_{21} 45$ (B) 3 (C) 9 (D) $\log_{231} 2079$ (E) N.O.T.

45. If $\csc\theta=-2.6$ and $\pi<\theta<\frac{3\pi}{2}$, then the value of $\sec\theta$ is

(A) $\frac{12}{13}$ (B) $-\frac{13}{12}$ (C) $-\frac{13}{5}$ (D) $\frac{12}{5}$ (E) N.O.T.